Strategic Ambivalence Among Alternative Investment Choices under Uncertainty

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Abstract

We develop a real options framework that addresses strategic ambivalence (or hesitancy) in choosing among investment or divestment alternatives under uncertainty. Our framework identifies and characterizes a region of 'strategic ambivalence' where the firm may rationally delay making a decision, even when choosing one alternative currently has high positive net present value (NPV) and/or appears superior to the alternative. The extent of the strategic ambivalence region is driven by scale differences in project-specific costs, expected growth rates and volatilities, and correlation among the competing alternatives. We discuss several business settings in which strategic ambivalence may occur, such as market entry or exit , new product development, geographic capacity expansion, and spin-offs. We also consider ambivalence with sudden random resolution of uncertainty.

1. Introduction

Investment decisions revolve around choosing among competing alternatives, be it choosing among which products to produce, which capital investments or R&D programs to undertake, the next phase of geographical expansion or go-to-market strategy, or exiting the market. All these decisions are made under conditions of uncertainty and often involve partially irreversible commitments making the stakes from taking a decision high. The need to account for uncertainty and the (partially) irreversible choice among alternatives is core in modern finance. Compared to traditional methods rooted in expected cashflows, the real options approach offers the possibility of delaying one's choice as yet another alternative. A classical insight of real options theory (Dixit and Pindyck 1994, Trigeorgis 1996) is that a decision maker making a partially irreversible decision should delay making that decision until the option value is "deep in the money." Standard real option models have not accounted, however, for a delay choice among concrete alternatives. In this paper, we extend this framework to rationalize how managers navigate such choices among specific investment alternatives and analyze the economic forces driving strategic ambivalence in decision-making processes under uncertainty.

Strategic ambivalence may often arise when the NPV from one alternative exceeds the NPV from another, but marginally so. In a dynamic setting, a firm may delay making any decision for some period even though (at least) one alternative is worth undertaking on its own right (it is 'deep in the money'). This brings out the notion of ambivalence, previously studied in the field of organization science (see Ashforth et al., 2010; Rothman et al., 2017; and references therein). Although typically viewed as something to be avoided, Rothman et al. (2017) highlight the beneficial role of ambivalence in enhancing flexibility and when management moves from a disengagement state towards commitment to a specific product or process. Our real options framework confirms that intuition of Rothman et al. (2017) by rationalizing strategic ambivalence.

Our basic setup concerns a context where the firm chooses between two alternative projects, one generating a lower gross expected cash-flow value at a lower cost and another generating higher gross expected value but at a higher cost. We demonstrate that this setup captures a variety of business settings involving the choice of investment scales (capacity), new products, locations for geographical expansion, or exit routes. For example, when choosing the scale of an investment, economies of scale play a critical role on whether a firm may opt for high scale when faced with favourable demand scenarios, but lower scale otherwise. Relatedly, in the context of geographical expansion, a firm might choose to expand in large scale when investing in high-demand area involving a more dense consumer base or focus on a lower-demand area if setup costs are lower. In new product development, significant innovation may bring higher upside, while low-innovation products may offer lower costs due to reduced complexity in development, while providing higher reliability building on existing technologies. Importantly, our model captures the above strategic trade-offs in a real options setting enabling the study of the effect of uncertainty and the decision timing in investing in strategic alternatives.

Our real options setup has important implications regarding ambivalence in decision making. In all the above cases, if the firm must commit to the low-cost/low-value project or the high-cost/high-value one at a given time, it will choose the first project in low states and the second project when gross project value is high.[1] But if the firm can delay the decision among these close alternatives, we posit that a region of strategic ambivalence may arise at

[1] We focus on cases of irreversibility, in which the firm cannot "undo" previous decisions or decide in multiple stages. The key insights that the choice among alternatives leads to strategic ambivalence would remain in cases of partial reversibility or multistage decisions (provided the payoffs of the various strategic routes cross at one point at least).

intermediate project values, leading the firm to delay committing on the scale of production, geographic expansion or new product development until more information is obtained. The existence and range of this strategic ambivalence region depends on key factors such as the volatility of each alternative, their growth and discount rates, and other project-specific parameters. In particular, high volatility makes the firm delay the decision at a future time when it is more clear which of the two alternatives is best (e.g., the large-scale project yielding higher returns than the low-scale alternative if the market is large).

We further consider several extensions of the main setup. We consider the case where management faces high initial uncertainty that is reduced at random time in the future. This additional layer of uncertainty (regarding the underlying process dynamics) exacerbates strategic ambivalence. In contrast, when the firm anticipates only a brief period of initial high uncertainty, the ambivalence is reduced, and the firm focuses on timing the exercise of the high-scale alternative (which occurs in high demand states).

Second, we incorporate multiple stochastic variables while accounting for the correlation among the two alternative project values (e.g., foreign investments in different national economies). We show that lower correlation among the alternatives results in a more valuable opportunity to choose the best alternative and an a wider ambivalence region.. This is because a lower correlation increases total relative volatility and the range of extreme outcomes, and hence postpones the firm's commitment to a particular investment choice. We also bring out the importance of individual project characteristics (growth rates and volatilities) showing that a firm may decide to postpone adoption of one alternative if the other offers higher upside potential (e.g., due to higher growth and/or volatility). Coupled with the effect of correlation, this setting can help explain why strategic ambivalence may help gain a better understanding in many business situations where firms face choices involving different market dynamics (e.g., products focusing on different geographical regions or different technologies) and various exit routes such as spin-offs or equity carve-outs.

2. Related literature

Our work has implications for firm's capacity choice, R&D policies, geographical expansion choices and exit routes. Below we thus review related literature focusing more on real options work applied to these areas. Trigeorgis and Reuer (2016) review real options theory in strategic management research, examining how it can shed light on the critical challenges managers face between maintaining flexibility and investment commitment. We also review some works on option theory that share related methodological features.

2.1. Capacity choice

An extensive literature on real options focuses on the optimal timing of capacity choice (e.g., see Dangl, 1999; Bensoussan and Chevalier-Roignant, 2019; and Huberts et al., 2015 for a review). We depart from this literature which focuses on a continuum of capacity choices, to study strategic ambivalence in the choice between a finite set of alternatives (e.g., high versus low capacity). Importantly, we demonstrate how to estimate the parameters of the model based on the economies of scale and we also capture new features such as finite investment horizon and multiple uncertainties.

2.2. Location choice and multinationals

Kogut and Kulatilaka (1994) study a multinational corporation which derives value from the opportunity to benefit from uncertainty through the coordination of subsidiaries which are geographically dispersed. Using real options, they model this coordination as the operating flexibility to shift production among manufacturing plants located in different countries. Real options theory has also been widely applied in incorporating risk and managerial flexibility when evaluating FDI projects (Chi et al., 2019). Rivoli and Salorio (1996) examine the timing of FDI decisions under uncertainty, focusing on the "option to wait." Similarly, Li and Rugman (2007) examine whether an MNE should establish a new subsidiary abroad ("location choice") and how it should approach this ("market-entry mode"). Li and Rugman (2007) differentiate between three market entry modes: "exporting/licensing," "joint venture," and "wholly owned subsidiary," showing how market volatility and option exercise costs influence both location and entry mode decisions in foreign markets. Alcácer et al. (2015) discuss the importance of learning by doing in shaping location choices in markets and how firms tend to collocate in local and global markets to preempt rivals.

In contrast to the above literature, our study focuses on the initial capital investment choice among locations. We capture the choice between low-scale/low cost location against a higher scale/higher cost one involving a higher potential albeit also more intense market competition. Our framework also captures such capital choices for the broader case involving investments in different, less than perfectly correlated asset locations.

2.3. R&D and new product development

McGrath and Nerkar (2004) posit that R&D investments create a portfolio of real options . They focus on how the scope of opportunity, prior experience, and competitive effects influence firms' propensity to invest in R&D.

Oriani and Sobrero (2008) leverage real options theory to show the effects of market and technological uncertainty on the value of R&D projects. In their analysis, market uncertainty primarily influences the existence of future growth opportunities, while technological uncertainty relates to firm survival in the face of technological change. In comparison, our models focus mostly on market uncertainty; in an extension we consider higher initial uncertainty regime, which may be due to technological uncertainties.

Gao et al. (2021) provides a framework where a product focusing on a higher level of innovation results in greater expected performance whereas a less innovative product balances the lower expected returns by requiring less resources and lower costs to develop. Our paper provides a real options valuation setting to evaluate such R&D choices and characterizes the decision regions depending on the value, growth potential and uncertainties of each alternative.

Klingebiel and Rammer (2014) provide empirical evidence that allocating resources to a broader range of innovation projects increases new product sales. Their analysis underscores the importance of building options and delaying the choice of narrowing focus in new product development. In comparison, our focus is on the commitment of irreversible mutual exclusive competing R&D choices with different potential driven by different characteristics (e.g., volatility, growth etc).

Building technological expertise is also crucial for success in new product development. Nerkar and Roberts (2004) find that new product introductions are more successful when a firm possesses superior technological knowledge, which is positively affected by accumulated technological experience, combinative capabilities, and complementary assets (e.g., market-related knowledge, product reputation, distribution channels, and customer contacts). Such complementary assets also depend on firm history with respect to product-market participation. In our model such prior experiences may affect the characteristics of the competing choices depending on where the firm has accumulated more experience.

2.4. Inertia or ambivalence in option theory

The existence of a phenomenon akin to strategic ambivalence has been examined previously in the literature on options, albeit without a complete characterization or without pointing the strategic implications. Boyle et al. (1989) were among the first to articulate that, with a choice between two stochastic variables, an inertia (delay) region may result when project values are nearly at the same level. Décamps et al. (2006) provide mathematical foundations (in case of perpetual American options) for the existence of inertia when choosing between projects of different scales. Bobtcheff and Villeneuve (2010) apply a similar methodology to price perpetual American options in the context of power generation capacity choice under uncertainty. Detemple and Kitapbayev (2020) study investments in mutually exclusive projects with different cost structures, focusing on the effect of operating leverage on investment decisions and assessing the impact of knowledge acquisition. Chevalier-Roignant (2024) extends the idea to study incumbents' strategic choice among acquiring a start-up or investing in organic growth. His work shows that incumbent's inertia, often viewed as a characteristic of complacency, may be rationalized based on strategic ambivalence. In contrast, this paper discusses a more general framework for the study of strategic ambivalence in different investment and exit strategy settings. We are also the first to consider a finite-horizon problem and to study the influence of model parameters on strategic ambivalence, including with multiple stochastic project values.

2.5. Modelling multi-asset uncertainty

Uncertainty can arise from various sources, as has been acknowledged early on in the options literature. Early contributions include Margrabe's (1978) model for (European) exchange options, Johnson's (1987) study for the European option on the maximum or minimum of several assets, and numerical methods works of Broadie and Detemple (1997), Barraquand and Martineau (1995), and Boyle et al. (1989). Martzoukos (2001) examined options on several stochastic variables to model exchange rate risk, while Martzoukos (2003) also accounts for multiple sources of jump risk. In contrast, our focus is on strategic decisions to characterize the possible ambivalence in choosing among multiple competing alternatives. Our paper provides new insights into firms' allocation of resources in capacity, new product development or multinationals' expansion in new geographical areas, as well as exit decisions.

3. Theoretical framework

3.1. Baseline model

The gross project value the firm can obtain from selecting either alternative is represented by a common uncertain factor V with initial value V_0. As standard in the real options literature, we assume that V follows a geometric Brownian motion (GBM) with drift parameter μ denoting the expected rate of value changes and σ (>*0*) capturing volatility. We assume the existence of a risk-free asset earning risk-free interest r (>*0*).

We assume that the firm chooses among two investment alternatives at or before a given maturity T. The high-scale project, indexed H, can be implemented at a cost $X > 0$, while the low-scale project, indexed by L, costs a fraction $b \in (0,1)$ of the cost for the high-scale project, but only allows to generate a proportion $a \in (0,1)$ of the value V. Choosing the best alternative at the time of investment $t <= T$ leads to the payoff $max\{V - X, a\,V - \,b\,X\}$. We assume $a > b$, so that, for values $V < V_{\star} := \,(a - 1)/(1 - b)$, the firm prefers the low-scale project to the high-scale project, and inversely. Section 2.3 explains how a variety of problems in capacity choice, R&D and new product development and geographical expansion fit this generic framework, providing the micro foundations for parameters a and b. The firm can choose the best project (indexed B for "best") at a time of its choice within the timeframe $[0, T]$. The point V_* is a point of indifference, with the NPVs close to this point being roughly equal. We start in the last period *T*, so the values are:

$$H_T = \max\,[V_T - X, 0],\;\; L_T = \max\,[aV_T - bX, 0],\;\; B_T = \max\,[H_T, L_T] \qquad (1)$$

Following Bellman's (1957) principle of optimality, if the firm has not decided on the best alternative by time T, it chooses the one with the highest value at that time. If $B_T = H_T > 0$ (resp., $B_T = H_L > 0$), then the decision is $D_T =$ H (resp., $D_T =$ L). If B_T=0, which happens if neither project is worth undertaking, the firm abandons its investment altogether, which we encode as $D_T = \mathrm{A}$.

Consistent with dynamic programming, to solve for the options' value, we move backwards in periods prior to last $t < T$, with values calculated as follows:

$$H_t = \max\left[V_t - X, \widetilde{H}_t\right], \quad L_t = \max\left[aV_t - bX, \tilde{L}_t\right], \quad B_t = \max\left[V_t - X, aV_t - bX, \tilde{B}_t\right] \quad (2)$$

where $\tilde{\imath}_t$ denotes the expected present value of each possibility $i \in \{H, L, B\}$. The firm chooses to exercise early project H if $B_t = V_t - X$, in which case $D_t = H$, else if $B_t = aV_t - bX$, $D_t = L$. If $B_t = \tilde{B}_t$, then the decision is to wait (that is, $D_t = W$). Recursively, one obtains the value of the options and the firm's decision at $t = 0$. In the results we also report the stand-alone option values H and L, along with the corresponding decisions the firm would have made if it held these as separate options. Appendix A provides a finite-horizon numerical approximation that solves this problem.

3.2. Baseline model predictions

This section characterizes the "strategic ambivalence" when a firm faces multiple strategic alternatives. Figure 1 shows the value functions of the various alternatives including for the option on the best or maximum (red line), the option values of stand-alone projects H and L (in blue and green lines, respectively) and the corresponding decision regions for various levels of project values V. The figure focuses on the decision region at $t = 0$. We provide predictions on how the decision regions evolve over time in subsequent sensitivity analysis.

[Insert Figure 1 here]

As expected, for low values of project value V, the decision to delay investment (i.e., wait, W) is optimal since the option on the maximum coincides with the value from stand-alone alternatives --which in this low region is simply to postpone investment. After a certain threshold value, the low-scale project L becomes the optimal choice (hence the red line showing the option value on the max coincides with the green line of the option on the L project). Importantly, at a certain range of high project values, the firm's optimal decision is to postpone investment in either alternative. In this region, the red line showing the value of the option on the maximum of the two alternatives exceeds the value of either alternative, implying there is strategic value in delaying one's commitment to make an investment choice. Finally, in a region of very high project values, the firm's optimal decision is to exercise the H scale project.

How are the firm's decisions affected by project uncertainty and by the growth prospects of the projects? To answer these important questions, we provide sensitivity analysis. The classical theory (Dixit & Pindyck, 1994) argues that the option value increases as volatility increases or when growth estimates are stronger. Furthermore, a higher discount (resp., growth) rate is generally believed to accelerate (resp., delay) one's investment (see Fig.5.5 in Dixit & Pindyck, 1994).

In panel A of Figure 2, we report the difference between the option on the maximum and the maximum of the stand-alone alternatives for two levels of volatility (Panel A) and for different growth rates (Panel B). This difference shows the additional value provided by having the choice to time the decision on the two alternatives compared to holding a stand-alone option on one alternative. A positive difference suggests benefits from waiting in choosing the best of the two alternatives. A higher volatility and project growth or a smaller discount rate (not shown for brevity) result in additional benefits from delaying the decision. At intermediate levels of volatility increase, we observe a shrinkage of the L region and appearance of a region of ambivalence (delay) at certain high values of V. In addition, the region involving H is further postponed. As volatility and growth increase, the region of ambivalence increases for higher σ (see Panel A) and for higher μ (see Panel B).

[Insert Figure 2 here]

We observe, however, that if either σ or μ gets at very high levels, the region of postponement increases, further making the low-scale project a superfluous choice, such that it is never adopted by the firm. Thus, in very high volatility or growth environments, the firm waits so long (out of caution) such that, by the time it makes its decision to invest, the market is so large that a low-scale approach is ill-advised. Under high uncertainty and high growth, the low-scale project is delayed and effectively is never adopted, with the ambivalence region shrinking while the optimal exercise threshold for the high-scale project is increased. We summarize the above results in the following proposition.

Proposition 1:

The value of holding an option to choose between two investment alternatives increases with higher volatility (σ) and growth rates (μ) or a lower discount rate (r), as it provides flexibility in timing the decision. At higher levels of volatility or growth or at a lower discount rate, firms tend to delay investment decisions causing an increase in the ambivalence region. However, at very high levels of volatility or growth or at a very low discount rate, firms tend to delay investment decisions significantly, ultimately favoring a high-scale project while rendering the low-scale project superfluous.

One distinction of our setting compared to earlier setups (Décamps et al., 2006; Chevalier-Roignant, 2024) is our focus on finite-horizon problems. Figure 3 characterizes the changes in optimal decisions over time until maturity $T = 5$. For compactness, we focus on a comparison of decisions early in the project's life ($t = 0$) compared to the case where the time to maturity approaches (maturity $t = 4$). Panel A uses our base case volatility, whereas panel B demonstrates how the decisions change over time.

In Panel A, with low to medium levels of volatility, when the project is in early stages, there is significant option value of delaying commitment to either alternative, so the option on the maximum differs over a wider range of values for t = 0 compared to t = 4. This is because total volatility $\sigma\sqrt{T-t}$ is higher when the option is further away from maturity so the option to postpone the decision is more valuable.[2] Thus, when the level of volatility σ is low to medium, there is an increase in the ambivalence interval at early stages, and a shrinkage of this interval as the option approaches its maturity. As maturity approaches, investment is more likely, either in the low or the high-scale project, depending on the demand level. In panel B, when project volatility is very high, the option to delay at initial project stages is even wider. This makes L superfluous and the firm delays investment and chooses to exercise H for high project values. In contrast, closer to maturity the firm's ambivalence in the two projects may rise again since within a short interval until expiration of the opportunity their values may substantially change due to high volatility. The time effect follows a pattern consistent with our previous observations about the effect of larger volatility: the ambivalence region increases for small to medium levels of volatility and diminishes for high levels.[3] We summarize the following proposition concerning the effect of time-to-maturity of the option on the ambivalence region.

Proposition 2:

The ambivalence region evolves over time in a finite-horizon setting. For relatively low volatility levels, early in the project's life, when the option to delay has higher value due to

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[2] The total variance is a function of σ and time to maturity. When σ is at low to moderate levels projects at their early stage imply higher total variance which is still not high enough to overshadow the ambivalence region (see panel A). With σ high when the project is at very early stage implies a significant higher variance which makes ambivalence disappear.

[3] Formally, the total variance is a function of both σ and time to maturity $\sqrt{T-t}$. When σ is at low to moderate levels projects at their early stage imply a higher $\sqrt{T-t}$ and hence total variance which is however still not high enough to overshadow the ambivalence region (see panel A). With σ already high, when the project is at very early stage implies a significant higher total variance which makes ambivalence disappear.

greater effective volatility, the ambivalence region is broader. This reflects greater caution in committing to either investment alternative. As maturity approaches, the ambivalence region shrinks, and the likelihood of investment in either the low- or high-scale project increases, driven by demand levels. For high volatility scenarios, the opposite is true: the delay region is wider at early stages, rendering the low-scale project unnecessary and favouring the high-scale project for high values of demand. In contrast, with high volatility the ambivalence region increases as maturity approaches.

[Insert Figure 3 here]

3.2.Applications to common problems in strategic management

We next provide a micro foundation for our basic framework in the context of common strategic problems involving capacity choice, R&D, geographical expansion and new product development.

3.2.1. Capacity choice between large and small scale projects

Real options models involving capacity choice, such as those of Kort and Huisman (2015) or Bensoussan and Chevalier-Roignant (2019), typically model demand as follows:

$$p_t = x_t {Q_t}^{\varepsilon} \tag{3}$$

where $-1 < \varepsilon < 0$ determines the elasticity of demand, which is $\left(\frac{1}{|\varepsilon|}\right)$. A higher $|\varepsilon|$ implies a more inelastic demand.[4] The demand shock x_t affecting the price per unit in eq. (3) follows a GBM where μ is the expected rate of change and σ is the volatility parameter. The demand shock x_t represents the relative strength of the demand. Within our setting, the firm chooses

[4] In capacity choice a linear function is also often used instead of the isoelastic function. This assumption does not change our arguments. For an isoelastic we note that since $|\varepsilon| < 1$ this implies that our focus is on $\frac{1}{|\varepsilon|} > 1$, i.e., an elastic demand where an increase in prices by 1% causes a more than 1% decrease in quantity. In line with previous literature, demand is assumed elastic since if demand were inelastic profits would tend to infinity as the quantities tend to zero.

between two levels of capacity, Q_H or Q_L, at a one-time investment cost of κQ_i^{η}, $i = H, L$, with the constraint $\eta > 1$ implying decreasing returns to scale.

Once the capacity level is chosen, revenues are given by $\pi_{it} = p_{it} Q_i = x_t Q_i^{\varepsilon+1}$. The NPV at investment for each alternative, net of capacity costs, is:

$$V_i^N = V_i - \kappa Q_i^{\eta} = \frac{x_t Q_i^{\varepsilon+1}}{r-\mu} - \kappa Q_i^{\eta} \tag{4}$$

From our base model we can determine parameters a and b as follows. First, define the value for project H to be $V = \frac{x_t Q_H^{\varepsilon+1}}{r-\mu}$. Since V is a constant times a GBM, from Ito's Lemma it also follows a GBM with the same drift and volatility as x. Also define the investment (in this case capacity) cost of the large-scale project to be $X = \kappa Q_H^{\eta}$. We can then find *a* as the relative value V_L/V_H and *b* as the proportion of capacity cost of L relative to H. Thus, parameters *a* and *b* that define project L are as follows:

$$a = \left(\frac{Q_L}{Q_H}\right)^{\varepsilon+1} \leq 1, \qquad b = \left(\frac{Q_L}{Q_H}\right)^{\eta} < 1 \tag{5}$$

Since $\eta > 1$ and $\varepsilon < 0$, we have that *a>b,* as in our baseline model. This implies that if $V > V_\star \coloneqq (a-1)/(1-b)$, the firm prefers project H to project L, and inversely. Again, a region of ambivalence may occur where the firm postpones investment in either choice to gain more information before making a strategic capacity choice.

3.2.2. Geographical expansion

Real options theory plays a key role in incorporating risk and flexibility into multinational investment decisions (cf. literature review). When a firm decides on a geographic expansion, the choice location of, say, a shop or a manufacturing plant affects the setup costs arising due to differences in real-estate or labour market conditions. Location also affects the level of demand due to differences in market size or consumer preferences. As an example, assume a coffee franchise wishing to expand in the downtown (with high demand *H*) or in the suburbs (with lower demand *L*). Setting up the shop downtown requires an investment *X*, but only a fraction $b \in (0,1)$ of this cost is needed in the suburb. If k_i denotes the square foot price in the location $i \in \{L, H\}$, then b is given by:[5]

$$b = \frac{k_L}{k_H} \in (0,1) \tag{6}$$

Suppose the firm faces uncertain demand, with demand size Q_H and Q_L in regions H and L: the population is more dense downtown so $Q_H > Q_L$. Despite the higher expected demand downtown, the firm will likely also face higher competition in that region. To capture this in a simple Cournot-like quantity competition, we can assume that competition increases quantity by a factor $c_H > c_L$. The firm then faces prices described by:

$$p_{it} = x_t(Q_{it}\, c_i)^{\varepsilon}, i = H, L \tag{7}$$

The firm's expected profit per period is $\pi_{it} = p_{it}Q_i = x_t(Q_{it}\, c_i)^{\varepsilon+1}$.This implies that the value of asset $V_i = \frac{x_t(Q_{it}\, c_i)^{\varepsilon+1}}{r-\mu}$ in $i = H, L$. This defines the ratio of L and H as:

$$\alpha = \left(\frac{Q_L c_L}{Q_H c_H}\right)^{\varepsilon+1} \in (0,1) \tag{8}$$

2.3.3. R&D and new product development

Earlier research has shown the use of real options in analyzing R&D and portfolios of investments (e.g., McGrath and Nerkar 2004, Huchzermeier and Loch, 2001). In the context of new product development, a firm typically determines the innovation level, choosing between an incremental or radical innovation. Gao et al. (2021) discuss a framework where a higher level of innovation results in greater project performance. Although it has lower market

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[5] If *X* includes the present value of labour costs it is also reasonable to expect that the costs at the outskirts of a city be lower than in the centre due to more labour supply shortages in the centre.

performance, the less innovative product requires less resources and cost to develop. In addition, its reliability can be achieved in a more cost-effective way since the firm already possesses the technological knowledge and experience to develop a reliable product. Gao et al. (2021) describe the cost function (see eq. (1) on p.257) that depends on the complexity of the product's innovation and the product's expected reliability depending on whether the product's innovation is low or high. This function can be used to estimate parameter b in our context with the less innovative product cost being a fraction of the cost of the high innovative product. Gao et al. (2021) also model product demand with customer valuation of the performance of low innovation product being lower than the high innovation product. Thus, similarly to our parameter a the low innovative product only obtains a fraction of the value of the high innovative one. However, different configurations of reliability may result in the value discount of low innovation products being less than the cost efficiency reduction of incremental low technologies resulting in cases where $a > b$.

4. Model extensions

This section introduces two key extensions. First, the firm may confront high initial uncertainty (due to uncertain R&D activity or initial fear of a location for expansion). The timeline within which this uncertainty will resolve is also uncertain. We utilize a regime-switching model that assigns a probability at each point in time for the resolution of uncertainty.

Previously, we assumed that the values of the alternative projects were perfectly correlated, which holds when investments target the same market, involve related technologies, or focus on regions influenced by similar factors. We here relax the assumption as it may not hold when a firm considers alternative projects with entirely different technologies and market dynamics.

4.1. High initial uncertainty about project value

Some investment opportunities may be subject to high initial uncertainty that may get resolved following the occurrence of an event outside the control of the firm, such as government regulations affecting the firm's revenues or costs or actively controlled by the firm through R&D (see e.g. Pindyck, 1993). This section considers a situation where the firm encounters significant initial uncertainty concerning a project, R&D activities, or the selection of a geographical expansion location, without a definite timeline for when this uncertainty will be resolved. Uncertainty, initially at a high level σ_1, is partially resolved and falls to the level $\sigma_2 < \sigma_1$, at an uncertain time, which is assumed exponentially distributed with parameter $\lambda > 0$ (see Guo et al., 2005; Driffill et al., 2013, Chevalier-Roignant, Villeneuve, Delpech, and Grapotte, 2024). Determining the firm's investment strategy is more involved and, here, is solved numerically (though analytical results are available). The options of the second period (with lower volatility σ_2) are valued using the procedure described in Section 2.1. Appendix B provides details on how we determine the optimal investment strategy in the first period. We here focus on the key insights from the model.

Figure 4 shows the decision regions for various probabilities of exiting the high-uncertainty regime. Initially, when the probability of leaving the high-uncertainty regime is zero, we replicate the scenario of sustained high uncertainty discussed earlier (see Figure 2), as the likelihood of transitioning out of this state remains zero. In this scenario, due to persistent high uncertainty, the L region shrinks, prompting the firm to defer investment and pursue the H project only for high project values. An additional layer of uncertainty arises as the volatility potentially decreases from high to low levels, initially expanding the ambivalence region. As regime uncertainty intensifies, the firm further delays investment, shrinking the L region and deferring action on the H project. However, as the probability of resolving the initial uncertainty approaches very high levels (driven by the parameter λ), the ambivalence region disappears entirely, leaving the firm to either defer investment or commit to the H project when project value is high. We provide the following proposition.

[Insert Figure 4 here]

Proposition 3:

A small probability of resolving the high initial uncertainty (λ) delays investment and generates strategic ambivalence. However, as this probability increases—leading the firm to expect only a brief period of high initial uncertainty—the ambivalence region gets reduced. At this point, the firm concentrates on the timing of the high scale (H) project, committing only when project value is high.

4.2. Two correlated stochastic projects

Both projects above were assumed to be driven by a common factor (i.e., correlation was assumed to be one). This assumption is reasonable if the alternative investments are in the same market, R&D for new product development focuses on related technologies, or the geographical expansion targets different regions but is potentially influenced by the same factors (e.g., expansion within the same country or a market with similar consumer preferences). Uncertainty, however, sometimes arises from different sources. In the area of new product development, for example, a firm may contemplate choices with completely different technologies and potential value dynamics. A technology firm may explore two different project choices—developing a smartwatch or developing a Virtual Reality (VR) headset, which deploy different technologies and address distinct markets.[6] Appendix C describes the numerical approach used in this case.

[6] Wearable technology focuses on compact, energy-efficient design and integration with existing devices, while VR requires high processing power and display innovation. The markets for these products also exhibit different dynamics. The smartwatch market might be driven by consumer convenience emphasizing the integration with mobile ecosystems. In contrast, the VR market is driven by advancements in gaming, entertainment or

4.2.1. Strategic ambivalence with less than perfectly correlated assets

Figure 5 illustrates the sensitivity of firm value with respect to the correlation (ρ) between the values of different alternative projects or assets. First, the lower the correlation between asset values, the higher the value of the option to choose the best and hence the firm value. As the correlation approaches one, results converge to the case of perfect positive correlation, seen previously in Figure 1. The higher firm value associated with lower correlation reflects the higher option value of selecting the maximum of two values, a benefit that becomes more significant when assets are less correlated (e.g., Broadie & Detemple, 1997). This occurs because lower correlation leads to greater potential differences between the two project values, enabling the option holder to maximize its potential. The total variance is larger for lower correlation, increasing option value.

[Insert Figure 5 here]

Value differences are quite significant, with the maximum difference between the perfectly negatively correlated and perfectly positively correlated scenarios exceeding 34%. Notably, even cases with positive but partial correlation show substantial value differences compared to the perfectly correlated scenario. For instance, the case with $\rho = 0.4$ has a value that is approximately 11% higher than the perfectly correlated scenario. Further, decisions at t=0 may differ depending on the level of correlation. In the case of perfect correlation, the optimal decision at t=0 is to invest in the low scale project L, whereas for correlation less than one, the firm optimally delays investment in either alternative.

professional applications like training simulations. The market potential and risks are distinct for each project. For instance, the smartwatch may face competition from established brands in the wearables market, while the VR headset may compete in a niche but rapidly evolving market with different growth trajectories.

Figure 6 shows the (in)decision regions in the case of two stochastic variables for various combination of the gross value of the two projects. We summarize the decision regions and ambivalence for two stochastic variables in the following proposition.

[Insert Figure 6 here]

Proposition 4:

In the case of two stochastic project values, we characterize four decision regions based on the asset values:

1. ***Out-of-the-Money Region (W):*** *For low values of asset 1 and asset 2 (corner left region) the firm waits since both options are out-of-the money.*
2. ***Low-Scale Project Region (L):*** *In the region where V_1 is relatively low and V_2 exceeds a certain threshold level, the low-scale project L is undertaken (exercised). In this region, the high-scale project is seen to have a very low value to warrant consideration or even postponement.*
3. ***Strategic Ambivalence Region (W):*** *In the region where both V_1 and V_2 are at intermediary high levels both projects are valuable, and it is advantageous to delay exercising any option (the wait region is expanding to the right of graph).*
4. ***High-Scale Project Region:*** *When V_1 gets very high, the high-scale project is undertaken.*

Figure 7 confirms these regions through numerical simulations, further examining the effect of correlation.

[Insert Figure 7 here]

We observe a significantly larger region of ambivalence when the correlation is low. Intuitively, there is greater potential gain from waiting a bit longer to see if the spread of outcomes between the two projects—and thus the NPV of the best choice—increases for either alternative when the correlation is low. Importantly, the decision to postpone investment in the ambivalence region occurs even if one or both projects are highly valuable. For example, when $X_1 = 100$ and $X_2 = 20$ are used, the case V_1= 200 and $V_2 = 200$ implies that both projects are highly valuable, yet the firm postpones committing to either alternative. This is not the case when the correlation between the assets is positive (ρ = 0.6). Instead, with higher correlation, the firm would not expect significant upside potential from the option to choose the maximum of the two projects, making it optimal to exercise and undertake the best of the two alternatives (in this case, the low-scale project L). Similarly, when project 1 is very valuable compared to project 2 (e.g., $V_1 = 400, V_2 = 200$), the firm will postpone investment when the correlation is low (e.g., see ρ = −0.6), reflecting the flexibility to delay commitment when the spread of outcomes increases in the future, making project L more valuable. This is not the case when the correlation is high, in which case the firm would commit to project H. We summarize the following proposition regarding the effect of correlation between project values on the ambivalence region.

Proposition 5:

A lower correlation between projects increases the ambivalence region, as the firm sees greater potential benefits from delaying investment to observe whether the spread between project outcomes widens. This flexibility to wait persists even when both projects are highly valuable. In contrast, higher correlation reduces the value of delaying, as the potential upside from choosing the maximum diminishes, prompting the firm to commit earlier.

4.2.2. Projects with different risk profiles

As discussed in Section 3.2, the assumption perfect correlation between assets typically involves conditions such as decreasing returns to scale for different capacity choices, increased complexity and costs for larger-scale R&D, and higher input costs combined with stronger demand competition for geographical expansion in high-demand areas.

When alternative projects involve different products and market dynamics, and hence potentially different correlations, ambivalence may arise even if both projects exhibit similar NPVs, returns to scale, complexity, or input costs but differ in their future market dynamics. For example, consider a case involving new product development with the following characteristics for the two projects.

Project H: $X_1 = 100, \mu_1 = 0.01, \sigma_1 = 0.25$.

Project L: $X_2 = 100, \mu_2 = 0.01, \sigma_2 = 0.35$.

Further parameters assumed are $r = 0.05$, project maturity $T = 5$ and correlation between project values $\rho = 0.6$. This case can reflect a situation where the firm faces two competing products with neither project having an edge over the other in terms of NPV. However, the firm may choose not to exercise its investment option early, even if one project is becomes more valuable than the other, due to differing expectations of future dynamics.

Figure 8 presents the firm's decisions at $t = 0$ for various combinations of V_1 and V_2 highlighting the ambivalence region.

[Insert Figure 8 here]

Due to less-than-perfect correlation and different project risks, there appears to be room to delay investing in either project as indicated by the wide ambivalence region, even if one or both alternatives are highly valuable (for immediate investment). The firm may postpone investing due to being ambivalent even if project 1 is currently significantly more valuable than project 2 (e.g., the case $V_1 = 200, V_2 = 150$) due to project 2's high volatility, which signifies a high upside future potential. In such case, there is strategic value in waiting and not committing to the currently more valuable alternative. The following proposition summarizes the effect of different risk profiles of projects.

Proposition 6:

When projects have different risk profiles, even with similar NPVs or growth dynamics, ambivalence may arise due to variations in volatility and future growth prospects. A project with higher volatility offers greater upside potential, prompting the firm to delay investment even if the other project is currently more valuable. This delay reflects the strategic value of waiting to capitalize on future market dynamics. The ambivalence region is widened when the projects have less-than-perfect correlation, highlighting the firm's hesitation to commit prematurely, even when one project appears significantly better in the present.

4.2.3. Strategic ambivalence in exit strategies

So far, our analysis has focused on strategic ambivalence arising from investment choices among two investment opportunities (e.g., different products or geographical regions). However, firms may also contemplate when to implement exit strategies, such as spin-offs, equity carve-outs, or reorganization. For instance, a parent company may spin off a division or subsidiary into an independent company, distributing shares to existing or new shareholders.[7] In this case, the firm may face several options regarding which divisions to include in the spin-off, influencing the potential selling price.

We adjust our framework to consider possible exit strategies, now modelled as a put option. We focus on a parent company that has two possible exit options: strategy H, which pays X_1 by spinning off a line of business currently valued at V_1, or strategy L, which pays X_2 by spinning off a unit currently worth V_2. Due to uncertainty, there is value in choosing the

[7] This strategy allows the parent to stay focussed on core operations while allowing the shareholders to cash-out from a sale of a division or unit.

best timing for the decision and selecting the best of the two exit strategies. At maturity of the exit strategy, the payoffs are:

$$H_T = \max\,[X_1 - V_{1T}, 0],\;\; L_T = \max\,[X_2 - V_{2T}, 0],\;\; B_T = \max\,[H_T, L_T] \qquad (9)$$

We consider an example where $X_1 > X_2$ and $V_1 > V_2$ implying that the choices are either spinning off a large scale of business H or a lower scale part of business L.

Figure 9 shows the decision regions for these exit strategies at a specific point in time. The high-scale exit strategy dominates when both projects exhibit very low values, as the firm recovers more from the high-scale exit. The low-scale exit strategy becomes dominant at somewhat higher values of the high-scale project V_1. For very high project values, the firm postpones exit because it would need to spin off highly valuable projects at low selling prices.

Importantly, there is a region of strategic ambivalence where, despite both exit strategies being currently valuable, the firm will delay exiting altogether. For instance, when asset values are $V_1 = 25$, $V_2 = 25$, the firm would postpone exit even though both strategies have positive NPVs.[8] Despite the positive NPV of exercising one of the exit strategies, the optimal decision for the firm is to delay committing to either strategy in order to gather more information about their future values and make a wiser future decision in choosing the best alternative exit strategy.

[Insert Figure 9 here]

Proposition 7:

Firms face strategic ambivalence in exit strategies, delaying decisions between spinning off large-scale or smaller-scale assets to maximize future gains. High-scale exits dominate when

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[8] In this state the NPV of each alternative is as follows: $NPV_H = X_1 - V_1 = 100 - 25 = 75$ while $NPV_L = X_2 - V_2 = 60 - 25 = 35$ so the firm would commit to the H project if there was no option to postpone the decision.

both projects have low values whereas the low-scale exits at more valuable for elevated values of the high scale project. The ambivalence region reflects the firm's preference to wait, even with positive exit NPVs of either alternative, for more information to optimize the exit choice.

5. Conclusion

This paper leverages real options theory to characterize and rationalize the notion of "strategic ambivalence." We show that ambivalence may arise when a firm faces multiple investment choices under uncertainty and that in a certain region of project values it is optimal for the firm not to commit to a certain investment choice but delay the decision for more information that may allow choosing the best alternative in the future. This size of the ambivalence region depends on the degree of volatility or growth of project value and on the correlation among alternative strategic choices when these are correlated. Importantly, our finite horizon setting allows us to analyse how decision regions evolve over time over the relevant uncertainty resolution or finite decision horizon. demonstrating that ambivalence may be more pronounced in the early stages of an alternative project choice for low to intermediate levels of project volatility.

We further link model parameters to specific applications in firms' strategic choices involving the scale of capacity, new product development or geographic expansion, rationalizing managerial postponement of investment decisions even though some alternatives may be worth undertaking in their own right. We also consider the situation when a firm faces an initial phase of high uncertainty that may get resolved but the manager does not know it will be resolved. We show that introducing the likelihood of resolving a high initial uncertainty creates an ambivalence region where the firm would postpone either investment option, with ambivalence going away when the likelihood of resolving uncertainty is high. In this case the firm focuses on timing the implementation of the high-scale alternative rather than delaying exercise altogether. Finally, we discuss how our setup may apply for firm exit or divestment strategies, illustrating situations when a firm may postpone committing to an exit strategy to gain valuable information on which strategy is more valuable.

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Appendix A: Finite horizon numerical solution

We use a binomial model to solve the finite horizon problem described in the main text. A standard formulation of the lattice parameters for the up and down jumps and the up and down probabilities requires that $u = \exp(\sigma\sqrt{h})$, $d = \frac{1}{u}$, $p_u = \frac{\exp(\mu h)-d}{u-d}$, where $h = \frac{T}{N}$ with N denoting the number of steps used in the binomial tree.[9]

Within the binomial tree we keep track of various data at each node: value of the project (V), the value of the option to invest in the high-scale project, the value of the option to invest in the low-scale project, the option on the maximum and the best (to-go) decisions of the firm, denoted D, at each node on the tree between wait (D=W), invest in low-scale project (D=L), invest in high-scale project (D=H).

At the end node, the underlying factor takes values given by $V_T = V_0\ u^{N+1-i}d^{i-1}$ with $i = 1,2, \dots N+1$ (from the highest to the lowest values). We start in the last period T so that values are:

$$H_T = \max\,[V_T - X, 0],\ L_T = \max\,[aV_T - bX, 0],\ \ B_T = \max\,[H_T, L_T] \qquad \text{(A1)}$$

In the last period, if the firm has not decided on the best alternative by time T, the firm will choose the one with the highest value at that time. If $B_T = H_T > 0$ (resp., $B_T = H_L > 0$), then the decision is coded on the decision tree as $D_T =$ H (resp., $D_T =$ L). If B_T=0, which happens if none of the projects is worth undertaking in its own right, the firm abandons all strategic routes, which we encode as $D_T =$ A.

[9] The probabilities and up and down step is matching the continuous time Geometric Brownian of the asset value at each step on the tree. Cox, Ross and Rubinstein (1979) also develop probabilities where the entire tree dynamics is consistent with the GBM dynamics. Both approaches yield accurate option values and decisions, however the ones we use are simpler to calculate.

Following Bellman's (1957) principle of optimality and consistent with dynamic programming, to solve for the options' value, we move backwards in periods prior to last t <T, where the values are calculated as:

$$H_t = \max\left[V_t - X, \tilde{H}_t\right],\ \ L_t = \max\left[aV_t - bX, \tilde{L}_t\right],\ \ B_t = \max\left[V_t - X, aV_t - bX, \tilde{B}_t\right] \quad \text{(A2)}$$

where $V_t = x_0\, u^{(n-j)} d^{(j-1)}$ with $n = N-1, N-2, \ldots .1$ describing the tree steps going backwards from T-1 to period 0. In addition, $\tilde{\iota}_t = \left[p_u i_{t+dt,u} + p_d i_{t+dt,d}\right]\exp\left(-rdt\right)$ describes the expected present value of each option $i = H, L$ and B which weighs the corresponding value on the next lattice step if value shock goes up (u) or down (d). The firm chooses to early exercise project H if $B_t = V_t - X$ in which case $D_t = H$, else if $B_t = aV_t - bX$, $D_t = L$. If $B_t = \tilde{B}_t$, then the decision is to wait (that is, $D_t = W$). Recursively, one obtains the value of the options and firm's decision at $t = 0$. In the results we also report the stand-alone option values H and L, along with the corresponding decisions the firm would have made if it held these as separate options.

A simple numerical example using the baseline model

Figure A1 leverages a simple numerical example to illustrate this notion of strategic ambivalence. Figure A1, panel A shows the dynamics of the underlying project value (V), panel B the option values in the case where the firm decides on the best of the projects, while Panel B (resp., C) shows the option values of investing in the low-scale (resp., high-scale) project.

[Insert Figure A1 here]

Panel B shows that a manager may optimally postpone the choice between the two projects until further realization of the uncertainty in the value of the projects. Importantly, the decision to postpone investment holds in regions where both investments are valuable. More generally, the decisions regions, starting from low values of the project V are to wait (D=W), adopt the low-scale project (*D=L*), wait (*D=W)*, and eventually adopt the large-scale project (*D=H*) for high realization of the underlying economic driver. The decision to postpone investing in the lowest-value region reflects the fact that the projects may not currently be valuable enough to pursue. The option to wait is more valuable than exercising the option at present. For example, in the state where the value of the asset is (9,9), the firm postpones investment in either project since neither provides any significant positive value.

The second region where delaying takes place at higher values of V, highlighted in light blue on the tree, is what we call the region of 'strategic ambivalence.' In this region, the firm postpones investment in either project, even though both are highly valuable. For example, in the state (3,9) the project value V is 202.81 implying an NPV of the high-scale project equal to 102.81 and an NPV of the low-scale project equal to 101.68, however the optimal decision is to postpone investment in either alternative. This seems counterintuitive at first, but arises because it is more beneficial to gather additional information before committing to either project. This region is located in the neighbourhood of the point V*, where the NPVs are close to one another, but not in a very significant manner. Here, the option to wait reflects the firm's uncertainty about which of the two projects will prove the best in the longer-term (while from a static, NPV perspective, the choice seems obvious). Panel C and D shows that indeed at least one of stand-alone project values is valuable enough for immediate exercise within the strategic ambivalence region. In this sense, if the firm held these options independently it would pursue one alternative. However, when deciding on the maximum of the two projects the firms may pause for some more information to arrive before committing to either alternative.

Appendix B: Regime switching model

In this section we show how to value the options in the initial regime of high uncertainty. First, we define the new parameters of the binomial tree to reflect the volatility of regime 1 as follows: $u_1 = \exp(\sigma_1\sqrt{h})$, $d_1 = \frac{1}{u_1}$, $p_u(1) = \frac{\exp(\mu h)-d_1}{u_1-d_i}$, where $h = \frac{T}{N}$. At the end node, the underlying factor takes values given by $V_T(1) = V_0\ u_1^{N+1-i}d_1^{i-1}$ with $i = 1,2, \dots N+1$ from the highest to the lowest values.

To value the options in regime 1, we start in the last period *T* so that values are calculated as follows:

$H_T(1) = \max\,[V_T(1) - X, 0]$,$L_T(1) = \max\,[aV_T(1) - bX, 0]$,$B_T(1) = max\,[H_T(1), L_T(1)]$

(A3)

Thus, in the last decision node in regime A, if $B_T(1) = H_T(1) > 0$ (resp., $B_T(1) = H_L(1) > 0$), then the decision is coded on the decision tree as $D_T(1) =$ H (resp., $D_T(1) =$ L). If $B_T(1)$ = 0, the firm abandons all strategic routes, which we encode as $D_T(1) =$ A.

Using dynamic programming, we move backwards in periods prior to last *t* <*T*, where the values are calculated as follows:

$$H_t(1) = \max\left[V_t(1) - X, \lambda dt\widetilde{H}_t(2) + (1-\lambda dt)\widetilde{H}(1)\right] \tag{A4a}$$

$$L_t(1) = \max\,[aV_t(A) - bX, \lambda dt\tilde{L}_t(2) + (1-\lambda dt)\tilde{L}(1)] \tag{A4b}$$

$$B_t(1) = \max\,[V_t(1) - X, aV_t(1) - bX, \lambda dt\tilde{B}_t(2) + (1-\lambda dt)\tilde{B}(1)] \tag{A4c}$$

where $V_t(1) = V_0\ u(1)^{(n-j)}d(1)^{(j-1)}$ with $n = N-1, N-2, \dots .1$ describing the tree steps going backwards from *T*-1 to period 0. $\tilde{\imath}_t(j) = \left[p_u i_{t+dt,u} + p_d i_{t+dt,d}\right]\exp(-rdt)$ describes the expected present value of each option $i = H, L$ and B which weighs the corresponding value on the next lattice step if value shock goes up (u) or down (d) under regime $j = 1,2$. The expected continuation value depends on whether the firm enters regime 2 with probability λdt or remains in regime 1 (with probability $(1 - \lambda dt)$). The firm chooses to early exercise project H if $B_t(1) = V_t(1) - X$ in which case $D_t(1) = H$, else if $B_t(1) = aV_t(1) - bX$, $D_t(1) = L$. If $B_t(1) = \lambda dt\tilde{B}_t(2) + (1 - \lambda dt)\tilde{B}(1)$ then the decision is to wait, $D_t(1) = W$. Recursively going backwards one obtains the value of the options and firm's decision at $t = 0$ within regime 1.

Appendix C: Multiple correlated uncertainties

In this appendix we focus on extending our framework to incorporate assets with different correlations. Our approach uses the binomial framework of Boyle et al. (1989) developed to model multiple assets. We consider two assets V_1 and V_2 with corresponding growth rates μ_1, μ_2, volatilities σ_1, σ_2 and a correlation ρ. Each asset can move up or down depending on its volatility on the tree as follows: $u_i = \exp\,(\sigma_i\sqrt{dt})$, $d_i = \frac{1}{u_i}, i = 1,2$. At each step in the binomial tree the two assets can take four possible moves: $1\!: (u_1, u_2), 2\!: (u_1, d_1), 3\!: (d_1, u_2), 4\!: (d_1, d_2)$. Boyle et al. (1989) define the probabilities of these possible states, which depend on the correlation between the two assets, as follows: below:

$$p_1 = -\,0.25\ \ (1\ +\ \rho\ +\ \sqrt{dt}\,((\mu_1\ /\ \sigma_1)\ +(\mu_2\ /\ \sigma_2)))$$

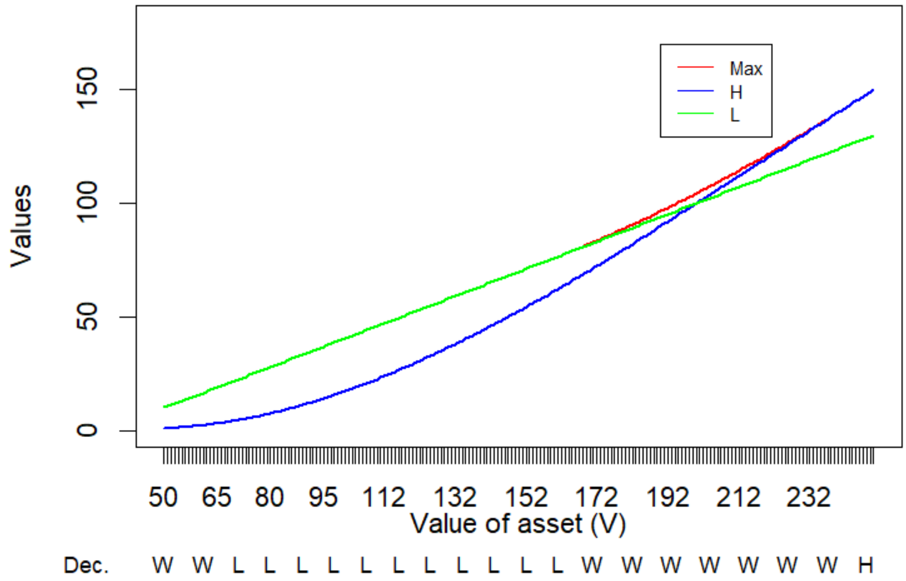
$$p_2 = -\,0.25\ \ (1 -\ \rho\ +\ \sqrt{dt}\,(\mu_1\ /\ \sigma_1) - (\mu_2\ /\ \sigma_2)))$$

$$p_3 = -\,0.25\ \ (1 -\ \rho\ +\ \sqrt{dt}\,(-(\mu_1\ /\ \sigma_1)\ +(\mu_2\ /\ \sigma_2)))$$

$$p_4 = -\,0.25\ \ (1 +\ \rho\ +\ \sqrt{dt}\,(-(\mu_1\ /\ \sigma_1)\ -(\mu_2\ /\ \sigma_2)))$$

Similar to the one-dimensional case, we start from the last period and move backward, appropriately weighing each state by its probability. This approach allows us to account for the potential co-movement of assets that are less than perfectly correlated.

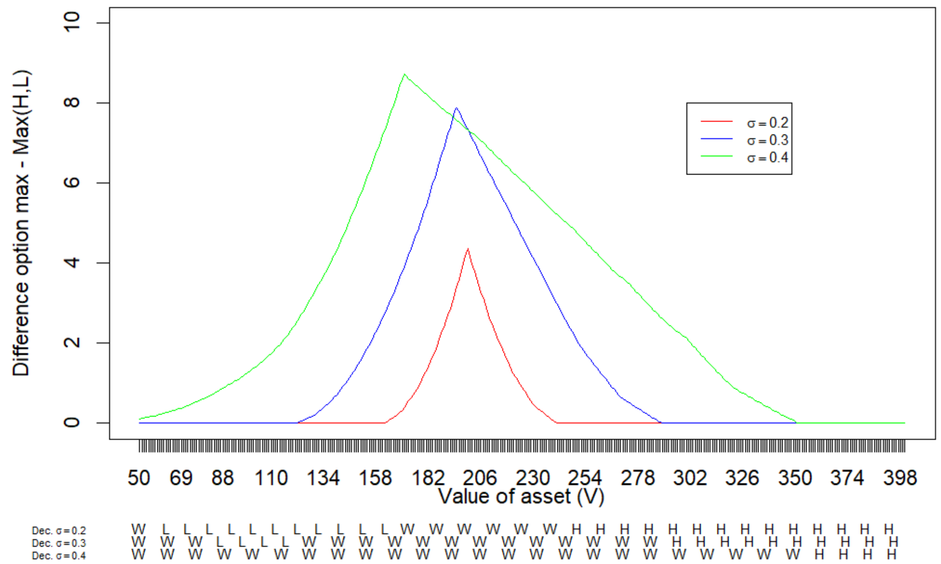
Figure 1. Decision regions and ambivalence



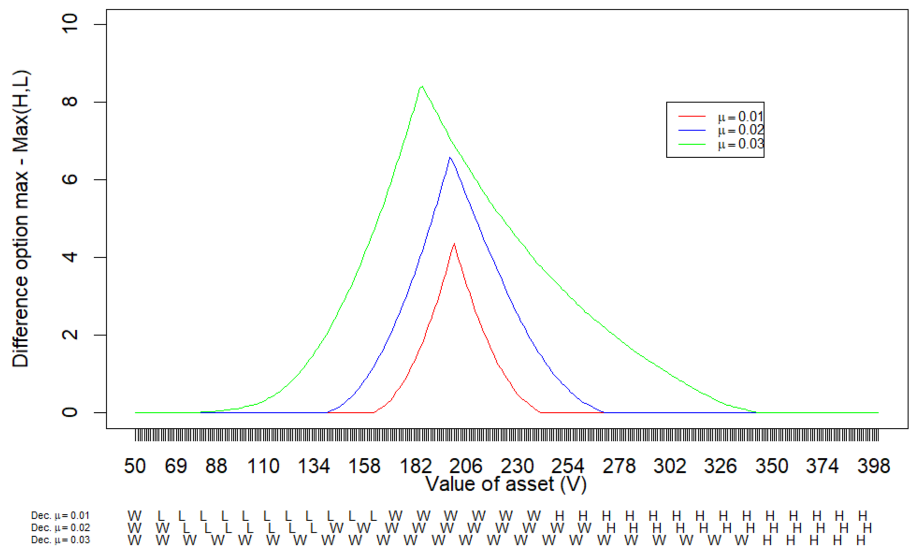
Notes: Parameter values are as follows: $V_0 = 100$, $X = 100$, $r = 0.05$, $\mu = 0.01$, $\sigma = 0.25$, T = 5, N = 100, a = 0.6, b = 0.2. The figure shows the decision value at t = 0 for different V values the option to invest in the H project (H), the value of the option to invest in the L project (L) and the value of the option on the maximum of the two projects (Max). The x-axis shows the value of the asset (V) and the corresponding decision of the firm relating the option on the max between delay (W), exercise L (L) or exercise H (H). The "strategic ambivalence" is region in W region between L and H regions. In this region at least one of the projects is highly profitable, however the firm waits for more information.

Figure 2. Decision regions and ambivalence: sensitivity to volatility and growth

Panel A: Sensitivity with respect to σ



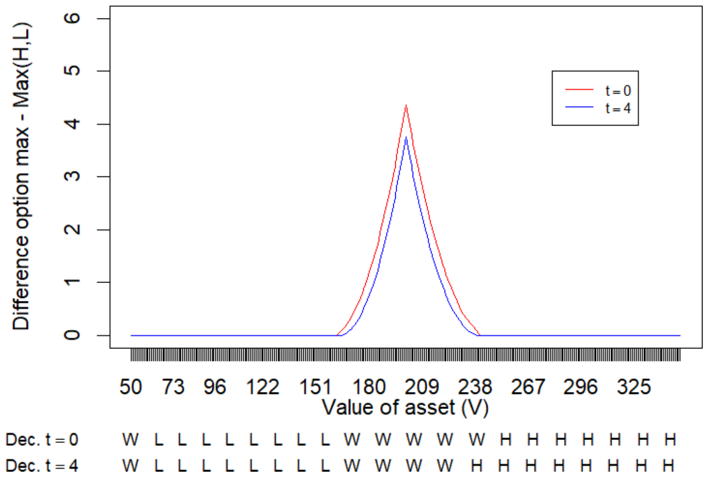
Panel B: Sensitivity with respect to μ



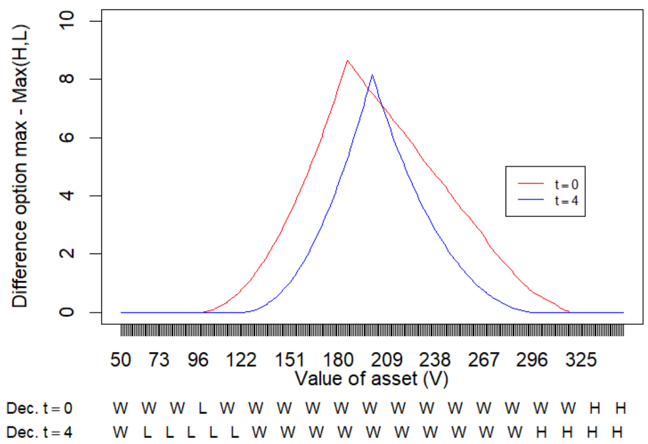
Notes: Bae case parameter values are as follows: $V_0 = 100$, $X = 100$, r = 0.05, μ = 0.01, σ = 0.2, T = 5, N = 100, a = 0.6, b = 0.2. The figure shows the difference between the value of the option on the maximum of the two alternatives H, and L minus the value of the maximum option to invest in the H project and L providing sensitivity with respect to σ and μ. The x-axis shows the value of the asset (V) and the corresponding decision of the firm relating the option on the max between delay (W), exercise L (L) or exercise H (H).

Figure 3. Decision regions and ambivalence: decision regions over time.

Panel A: Base case volatility

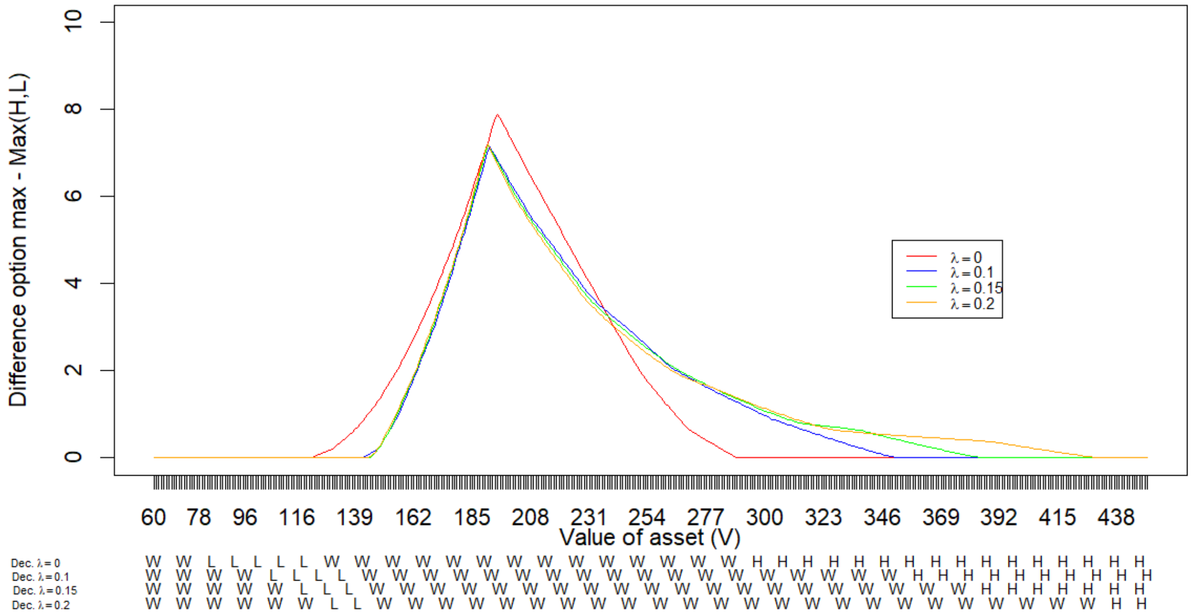


Panel B: High volatility



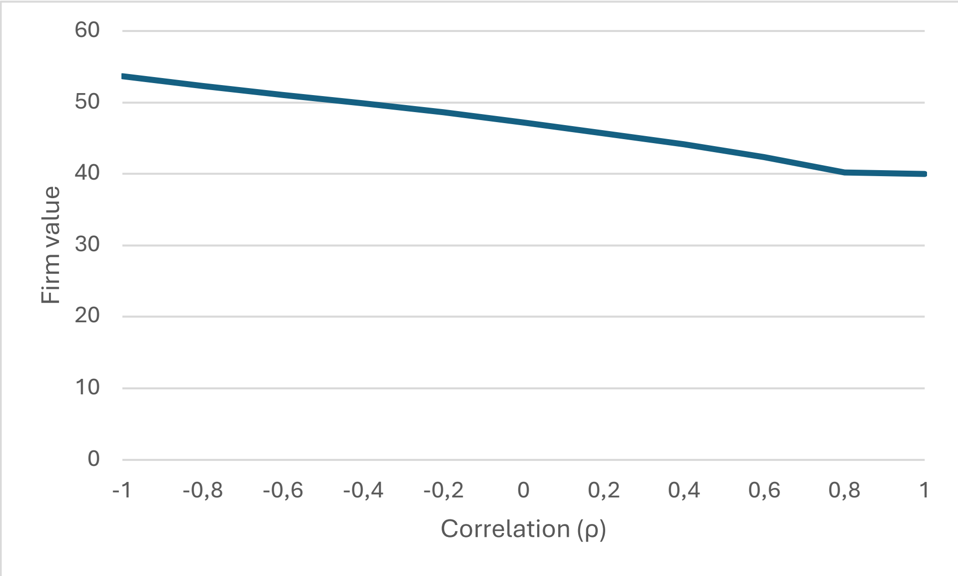
Notes: Base case parameter values are as follows: V_0 = 100, X =100, r = 0.05, μ = 0.01, σ = 0.2, T = 5, N = 100, a = 0.6, b = 0.2. The figure shows the difference between the value of the option on the maximum of the two alternatives H, and L minus the value of the maximum option to invest in the H project and L providing sensitivity with respect at t = 0 (assuming remaining time to maturity T = 5) and t = 4 (assuming remaining time to maturity T =1). Panel A uses base-case σ = 0.2, while panel B uses σ = 0.35. The x-axis shows the value of the asset (V) and the corresponding decision of the firm relating the option on the max between delay (W), exercise L (L) or exercise H (H).

Figure 4. High initial uncertain regime



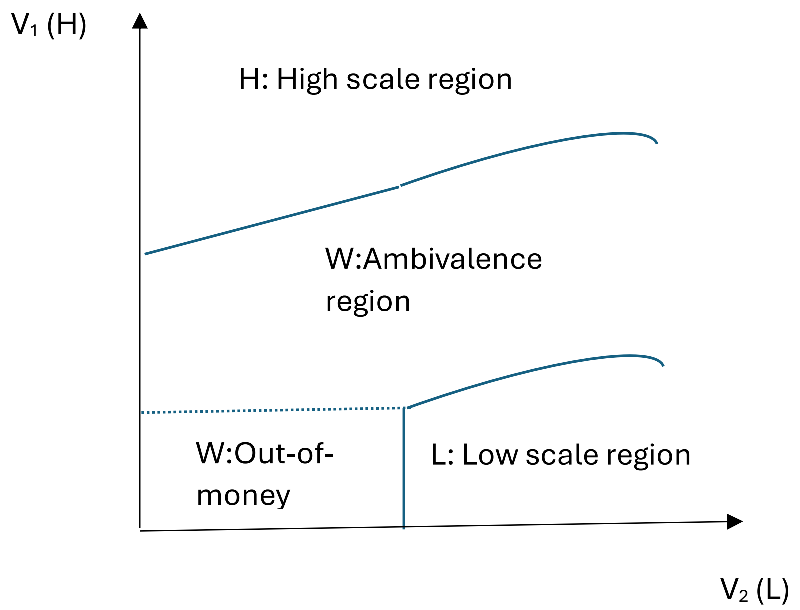
Notes: Parameter values are as follows: $V_0 = 100$, $X = 100$, r = 0.05, μ = 0.01, σ = 0.3, T = 5, N = 100, a = 0.6, b = 0.2 for different frequency of leaving in initial high uncertainty regime parameter. The figure shows the difference between the value of the option on the maximum of the two alternatives H, and L minus the value of the maximum option to invest in the H project and L providing sensitivity with respect to σ and μ. The x-axis shows the value of the asset (V) and the corresponding decision of the firm relating the option on the max between delay (W), exercise L (L) or exercise H (H).

Figure 5. Effect of correlation on firm value with project choices



Notes: Parameter values are as follows: $V_1 = 100$, $X_1 = 100$, $V_2 = 60$, $X_2 = 20$ r = 0.05, $\mu_1 = 0.01$, $\mu_2 = 0.01$, $\sigma_1 = 0\ .25$, $\sigma_2 = 0.25$, T = 5, N = 10. Sensitivity with respect to the correlation (ρ) between assets. We use the parameter values which are as in the baseline (see Table 1), albeit capturing the correlation between assets.

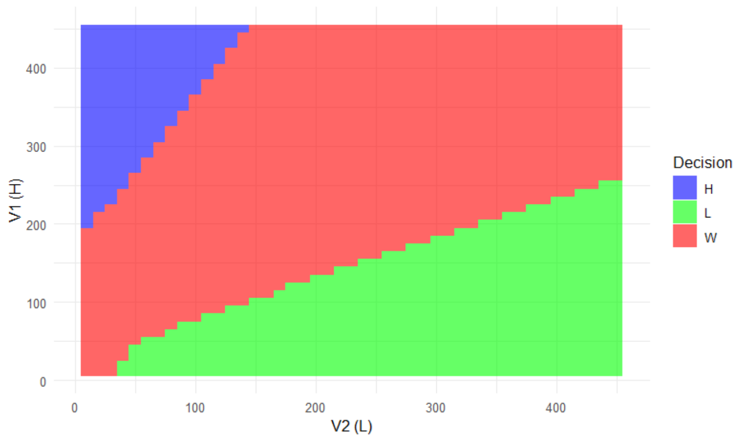
Figure 6. Decision regions with two stochastic project values



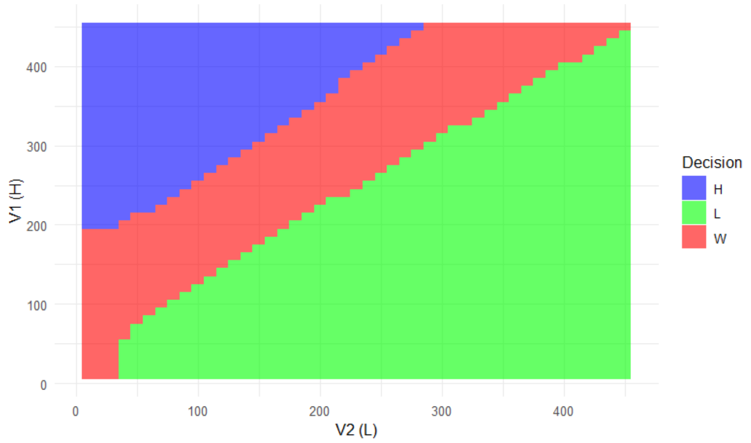
Notes: The figure shows the general decision regions for two stochastic projects.

Figure 7. Decision regions for two correlated assets

Panel A: Low correlation (ρ = -0.6)

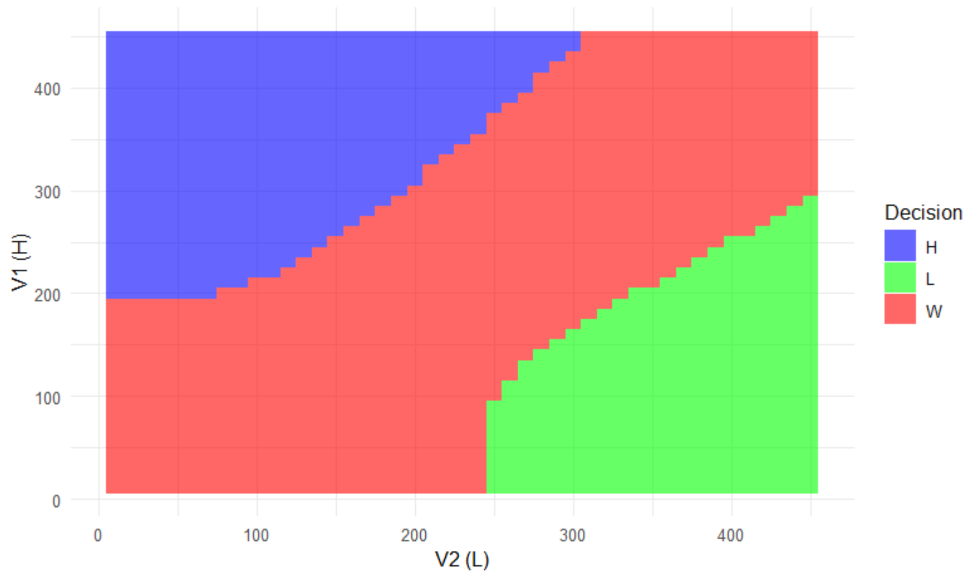


Panel B: High correlation (ρ = 0.6)



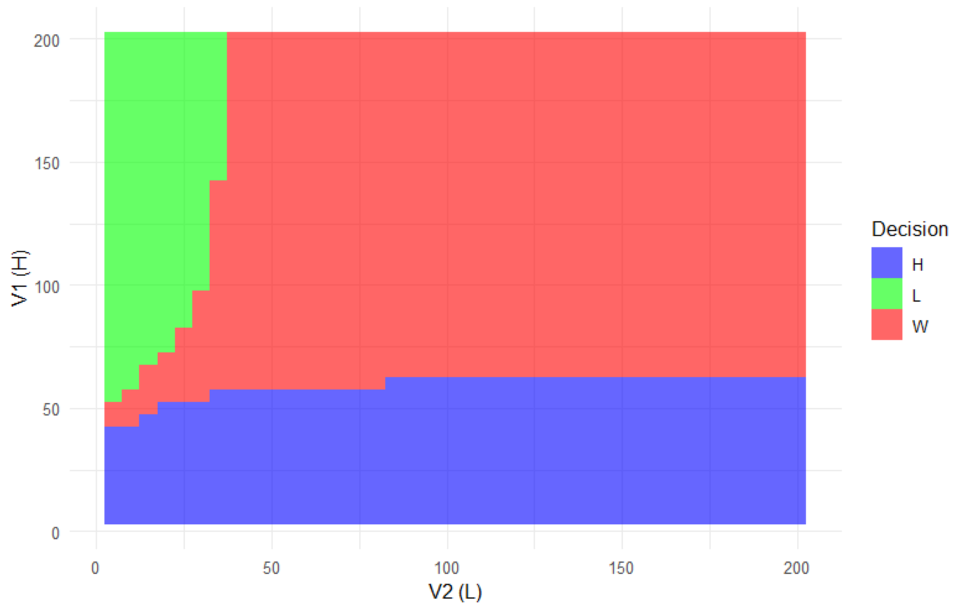
Notes: The figures show the decision regions for combinations of values of (V_1, V_2). Parameters used are $X_1 = 100$, $V_2 = 60$, $X_2 = 20$ r = 0.05, $\mu_1 = 0.01$, $\mu_2 = 0.01$, $\sigma_1 = 0.25$, $\sigma_2 = 0.25$, T = 5, N = 10. Panel A uses correlation ρ = -0.6 and Panel B using ρ= 0.6.

Figure 8. Projects with similar economies of scale but different upside potential



Notes: The figure show the decision regions for combinations of values of (V_1, V_2). Parameters used are $X_1 = 100$, $X_2 = 100$ r = 0.05, $\mu_1 = 0.01$, $\mu_2 = 0.01$, $\sigma_1 = 0.25$, $\sigma_2 = 0.35$, ρ= 0.6 , T = 5, N = 10.

Figure 9. Ambivalence in exit strategies



Notes: $V_1 = 100$, $X_1 = 100$, $V_2 = 20$, $X_2 = 60$ r = 0.05, $\mu_1 = 0.01$, $\mu_2 = 0.01$, $\sigma_1 = 0.25$, $\sigma_2 = 0.25$, ρ= 0.6 , T = 5, N = 10. The table shows the decisions at t = 0 for different values of V_1 and V_2 for the model of exit discussed in section 4.3.

Appendix Tables & Figures

Figure A1. A simple example of strategic ambivalence in decision making

Panel A: Binomial tree for underlying process (*V*)

State	1	2	3	4	5	6	7	8	9	10	11
1	100.00	119.34	142.41	169.95	202.81	242.03	288.83	344.68	411.33	490.86	585.78
2		83.80	100.00	119.34	142.41	169.95	202.81	242.03	288.83	344.68	411.33
3			70.22	83.80	100.00	119.34	142.41	169.95	202.81	242.03	288.83
4				58.84	70.22	83.80	100.00	119.34	142.41	169.95	202.81
5					49.31	58.84	70.22	83.80	100.00	119.34	142.41
6						41.32	49.31	58.84	70.22	83.80	100.00
7							34.62	41.32	49.31	58.84	70.22
8								29.01	34.62	41.32	49.31
9									24.31	29.01	34.62
10										20.37	24.31
11											17.07

Panel B: Option on the best of two alternative choices

State	1	2	3	4	5	6	7	8	9	10	11
1	40.00(L)	51.60(L)	65.45(L)	83.52(W)	108.40(W)	142.44(W)	188.83(H)	244.68(H)	311.33(H)	390.86(H)	485.78(H)
2		30.28(L)	40.00(L)	51.60(L)	65.45(L)	83.39(W)	108.10(W)	142.12(W)	188.83(H)	244.68(H)	311.33(H)
3			22.13(L)	30.28(L)	40.00(L)	51.60(L)	65.45(L)	83.10(W)	107.48(W)	142.03(H)	188.83(H)
4				15.43(W)	22.13(L)	30.28(L)	40.00(L)	51.60(L)	65.45(L)	81.97(L)	102.81(H)
5					10.22(W)	15.30(L)	22.13(L)	30.28(L)	40.00(L)	51.60(L)	65.45(L)
6						6.20(W)	9.92(W)	15.30(L)	22.13(L)	30.28(L)	40.00(L)
7							3.19(W)	5.62(W)	9.58(L)	15.30(L)	22.13(L)
8								1.18(W)	2.38(W)	4.79(W)	9.58(L)
9									0.16(W)	0.35(W)	0.77(L)
10										0.00(W)	0.00(A)
11											0.00(A)

Panel C: Option on stand-alone project L

State	1	2	3	4	5	6	7	8	9	10	11
1	40.00	51.60	65.45	81.97	101.69	125.22	153.30	186.81	226.80	274.52	331.47
2		30.28	40.00	51.60	65.45	81.97	101.69	125.22	153.30	186.81	226.80
3			22.13	30.28	40.00	51.60	65.45	81.97	101.69	125.22	153.30
4				15.43	22.13	30.28	40.00	51.60	65.45	81.97	101.69
5					10.22	15.30	22.13	30.28	40.00	51.60	65.45
6						6.20	9.92	15.30	22.13	30.28	40.00
7							3.19	5.62	9.58	15.30	22.13
8								1.18	2.38	4.79	9.58
9									0.16	0.35	0.77
10										0.00	0.00
11											0.00

Panel D: Option on stand-alone project H

State	1	2	3	4	5	6	7	8	9	10	11
1	20.11	31.37	47.80	71.01	102.81	142.03	188.83	244.68	311.33	390.86	485.78
2		11.08	18.30	29.49	46.20	70.13	102.81	142.03	188.83	244.68	311.33
3			5.21	9.26	16.08	27.19	44.50	69.95	102.81	142.03	188.83
4				1.88	3.65	6.99	13.14	24.05	42.41	69.95	102.81
5					0.39	0.86	1.87	4.09	8.91	19.44	42.41
6						0.00	0.00	0.00	0.00	0.00	0.00
7							0.00	0.00	0.00	0.00	0.00
8								0.00	0.00	0.00	0.00
9									0.00	0.00	0.00
10										0.00	0.00
11											0.00

Notes: Parameter values are as follows: $V_0 = 100$, $X = 100$, r = 0.05, μ = 0.01, σ = 0.25, T = 5, N=10, a = 0.6, b = 0.2. We use a horizon of $T = 5$ years which is a reasonable period for the firm on settling on the product or investment choice. The 5-year horizon has been used in other studies in new product development. For example, Pennings and Lint (1997) use a 5 year horizon to study Philips new product development in multimedia (see also Koussis, et al., 2013). Our choice parameters satisfy the inequality $a > b$, which reflects the intuition that the lower scale project L is expected to be chosen in intermediate market demand and value levels ($V < V_\star$, while project H dominates in scenarios of high market demand ($V > V_\star$). In light blue value we highlight the region of ambivalence in the option on the best of two choices. The same states are highlighted for the stand-alone values for comparison of decisions between the best of two choices and decisions on the stand-alone options.